CURRICULUM, PEDAGOGY AND BEYOND

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Graphs and Networks Below Level 10?

rob_money22@yahoo.com.au

The ‘Space’ Curriculum

What to ditch? – move?

Graphs and networks are now in the Year 10 to 12 curriculum.

Is this topic appropriate to earlier year levels?

Do we still need congruence axioms, angles on parallels?

What content best meets student needs and interests?

Year 6+ Simple Graphs

have no multiple edges

Five people are being paired up for mental arithmetic challenges against each other.

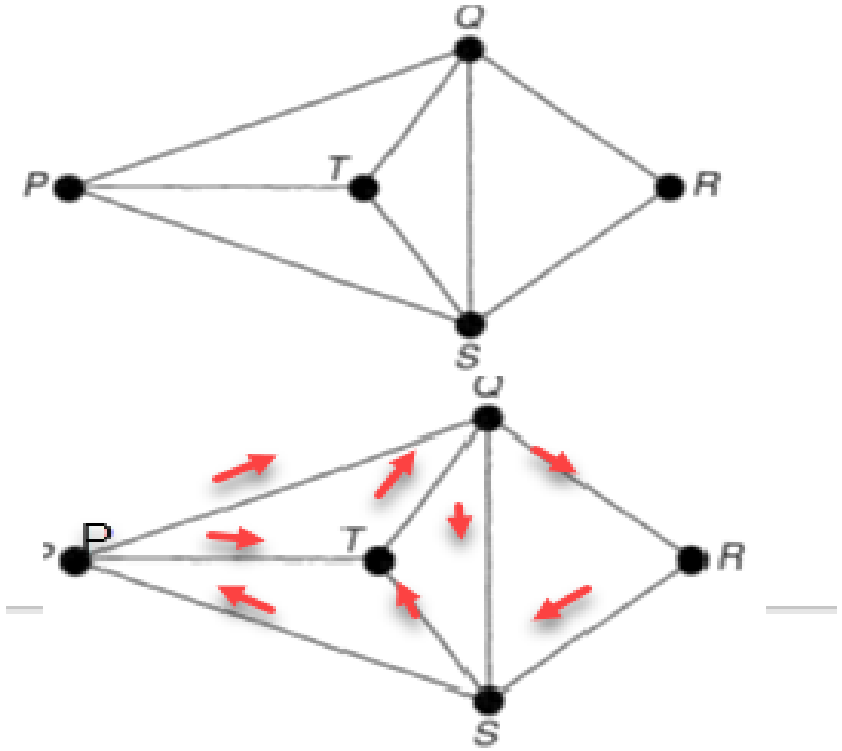
What challenges have yet to be conducted?

On the directed graph, what in-degrees will the vertices have if P beats R and R beats T?

How many edges are there on the complete graph on n vertices?

Complete Graphs

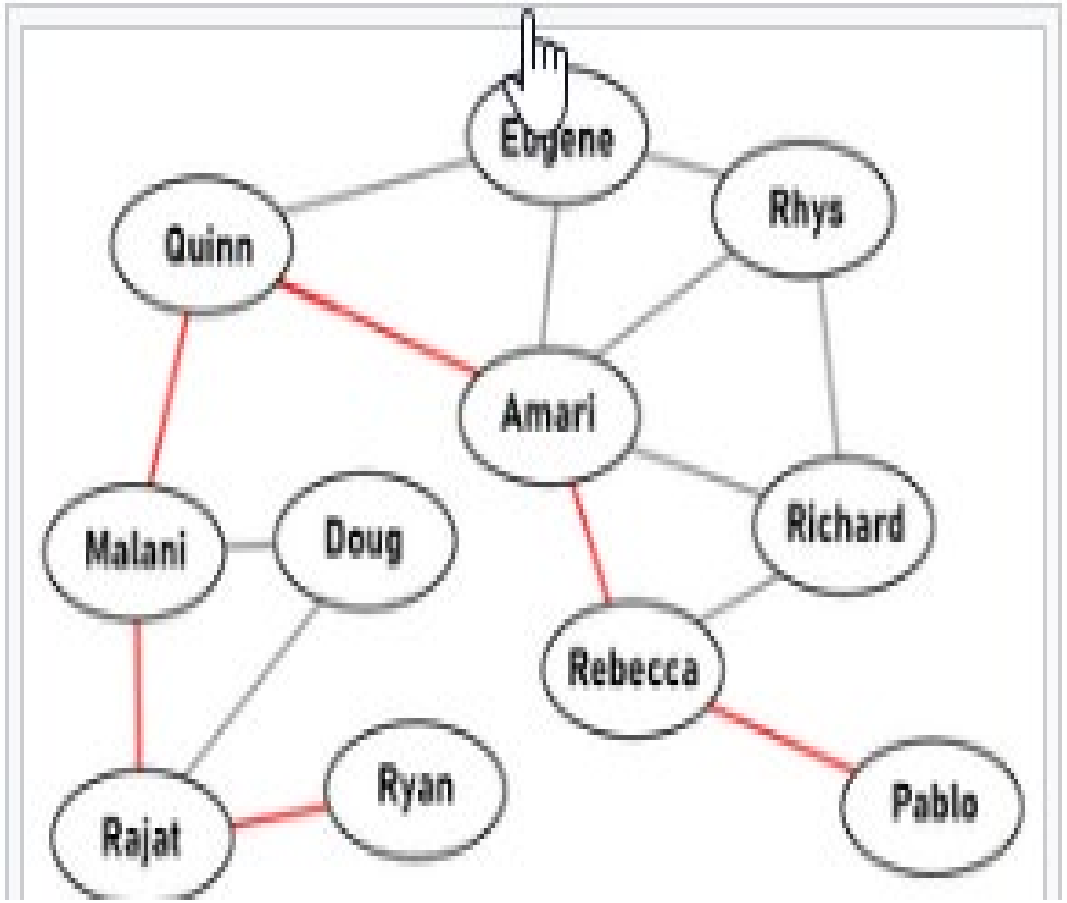
have all possible edges



a strongly connected directed graph

Social Network Connections

- Which student is most connected?
- Between which two students is there the greatest number of ‘degrees of separation’?



Year 7+ Directed Graphs

Each edge has a direction

- What is at the bottom of this food chain?
- What does each directed edge mean?

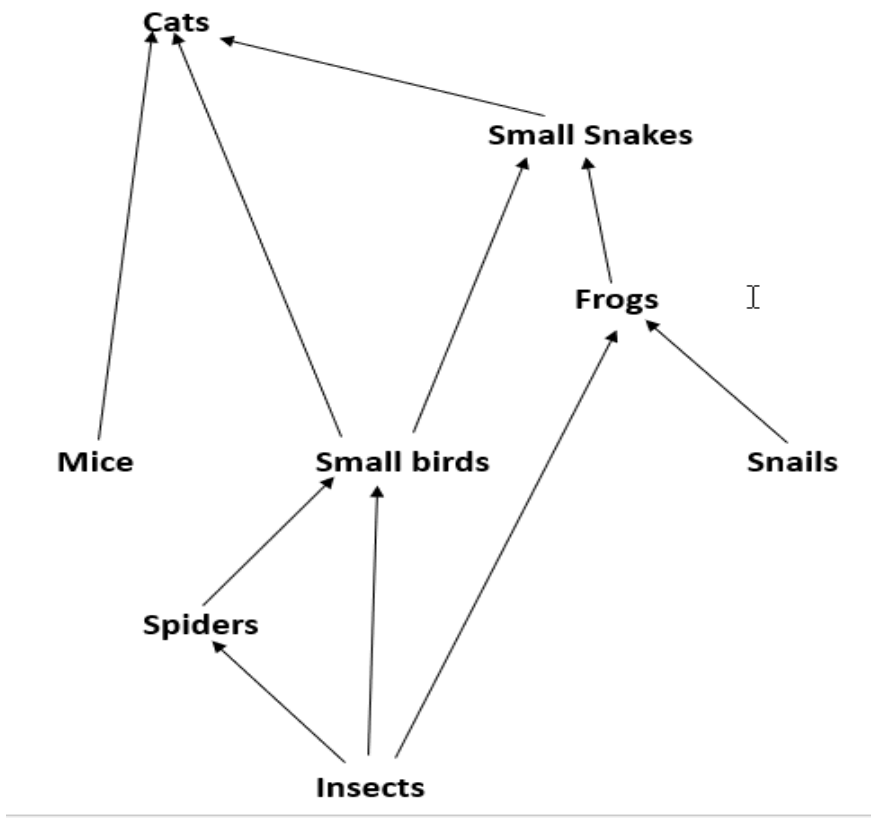
Uni-directional graphs:

Who do you work well with in group work?

Who do you go to for maths help?

Bi--directional graphs:

emails or SMS to and from



Year 7+ Bipartite Graphs

From ‘Source vertices’ to ‘Sink vertices’

Draw in 7 edges to represent the following.

Alby and Bet have visited NSW.

Charlie has visited NSW and SA.

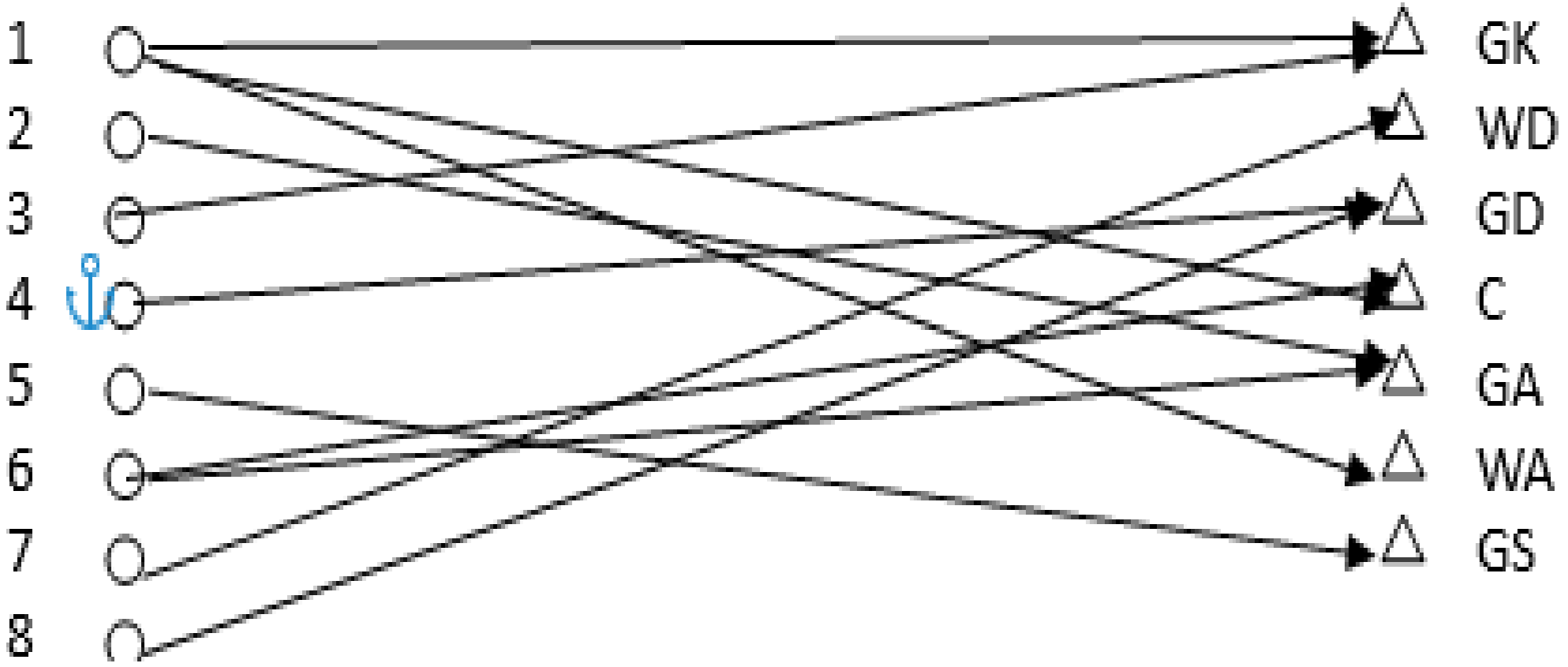
Dannie has visited NSW and QLD.

Evie has visited just QLD.

What other bipartite graphs might students make? e.g students vs sports that they play, electives they share or transport they use.

A Matching Problem - Netball

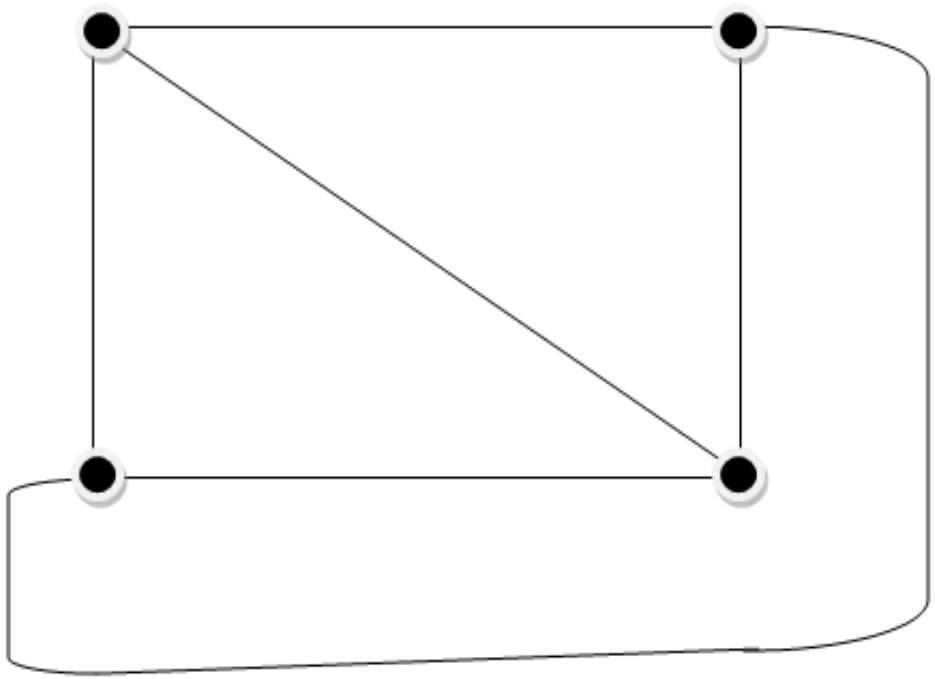
Can each position be filled?
If so which player misses out?



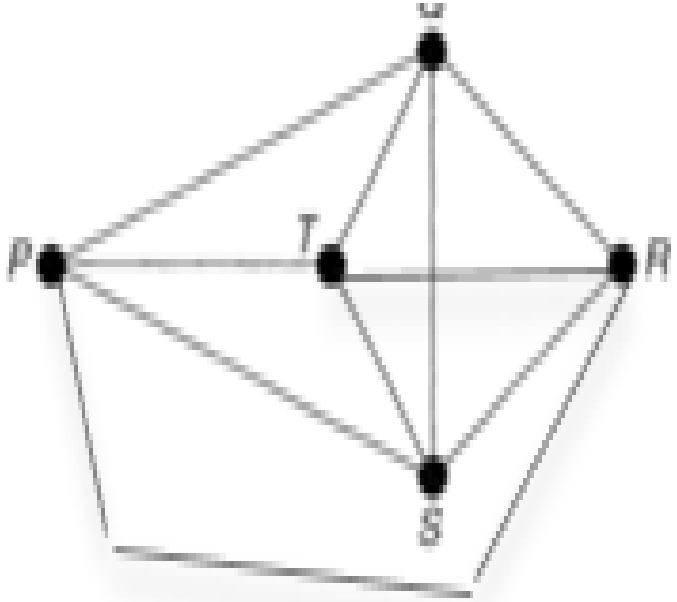
Year 8+ Planar Graphs

can be drawn with no overlapping edges

- The complete graph of 6 edges on 4 vertices, $\mathbf{K_4}$, is planar.



- The complete graph of10 edges on 5 vertices is not planar. Neither is any graph that contains this $\mathbf{K_5}$ as part of it

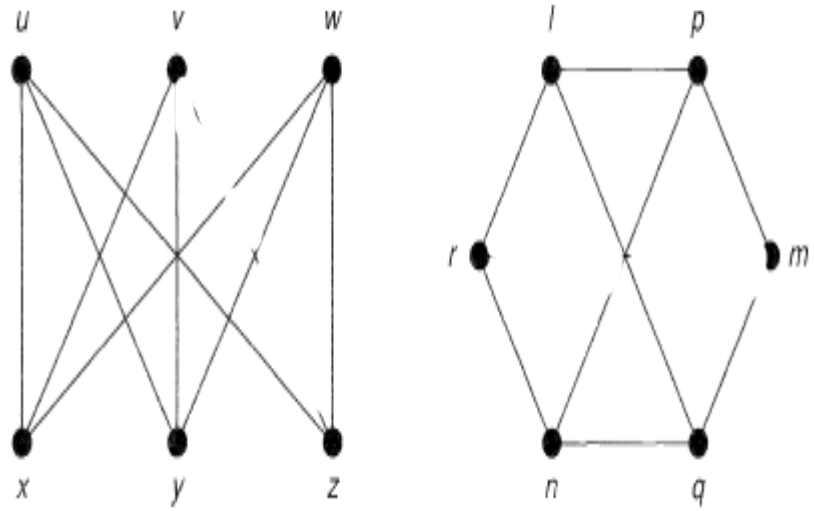


Year 8+ Which Graph is Planar?

Isomorphic graphs

Both! They are 'the same graph'.

- u - / v – m $w – n$
- $x – p$ $y – q$ z - r.
- These graphs are isomorphic and planar.
- For no intersections redraw pn.
- With one extra edge (vz or mr) we would have the complete bipartite graph $K_{3,3}$, which is non planar.
- A graph is planar iff it contains neither $K_{3,3}$ nor K_5



Year 8+: Euler's Rule and Map Colouring

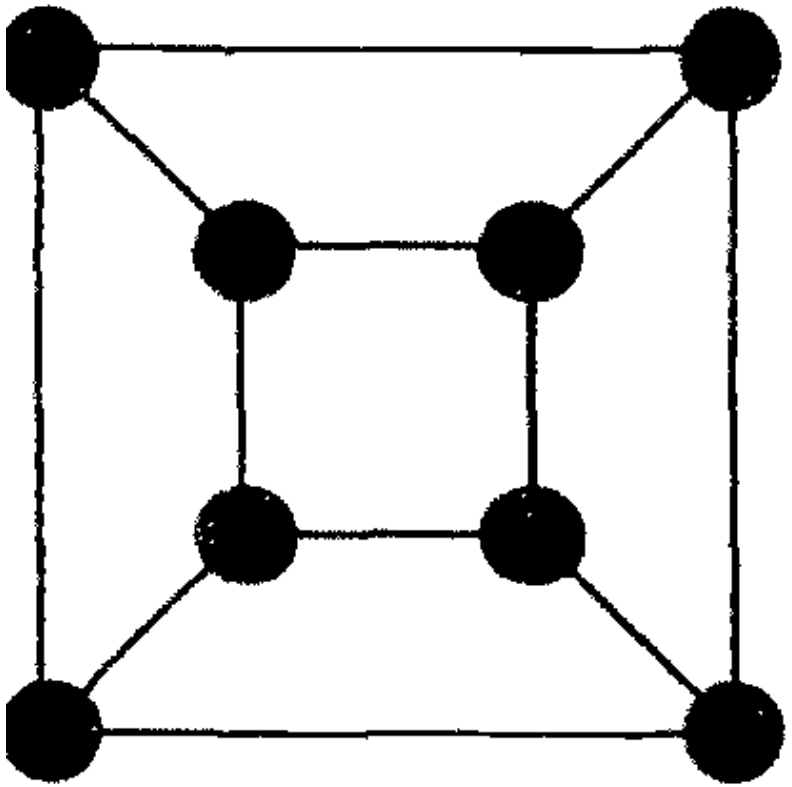
for convex polyhedra and all planar graphs

The outside of this Schlegel diagram represents the sixth face of the cube.

Prove V + F = E + 2. Starting with 1 + 1 = 0 + 2.

How many colours are needed here if adjacent faces need different colours?

Can you draw a graph for which four colours are required?

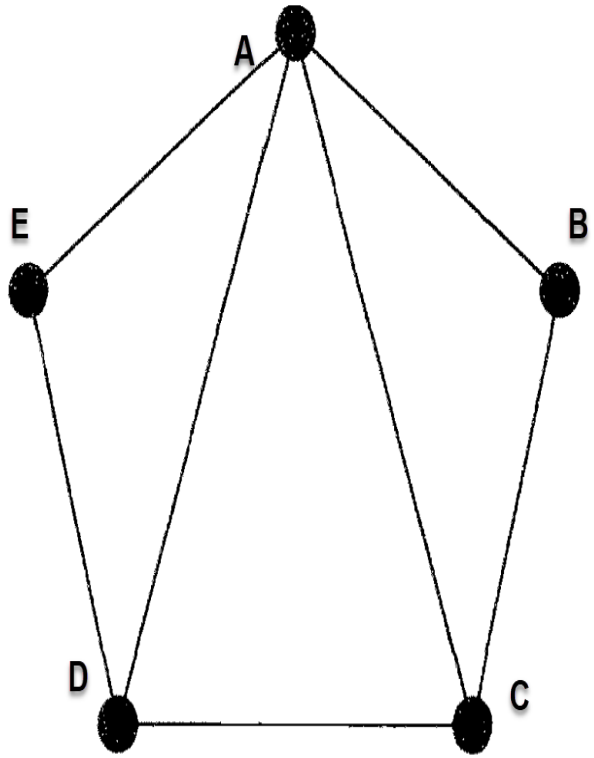


Year 9 Language: Trails & Paths

- A **walk** is a sequence of edges that are either adjacent or identical, e.g. ABA (length 2) and ABCAB (length 4).
- A **cycle** is a closed walk**(** last vertex = 1st**)** e.g. ABCDA
- A **trail** has **no repeated edges**

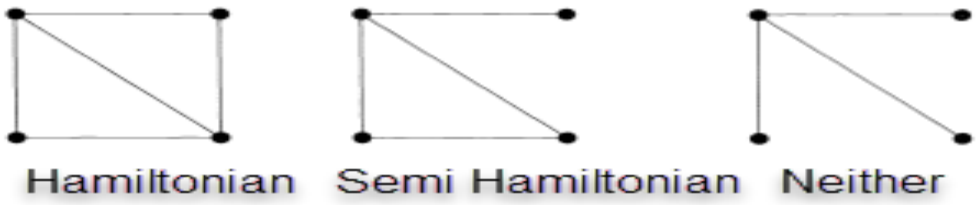
  e.g. ABCADE
- **Semi Eulerian trail (**all edges, once each**)** e.g. CBAEDACD
- **I**f edge CD was not there the graph would have the **Eulerian trail** ACBAEDA **(also** last vertex = 1st**) .**
- A **path** has **no repeated vertices**

  e.g. ABCD
- **Semi Hamiltonian path (**all vertices, once each**)** e.g. ABCDE
- **Hamiltonian path (also** last vertex = 1st**)** e.g. ABCDEA



Year 9: Hamiltonian Graphs

a cycle using each vertex just once



Knight's Tour problems were the first Hamiltonian problems studied – **Rudrata** in the 9th century.

A simple graph on 3 or more vertices is Hamiltonian if:-

- $\deg(v) \geq n/2$ for all vertices
- or if $\deg(v) + \deg(w) \geq n$ for all non-adjacent pairs

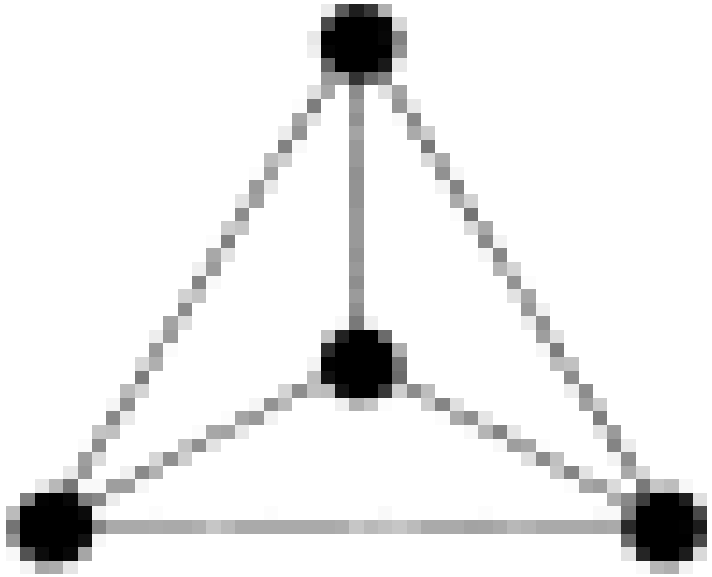
The converses do NOT apply.

Eulerian Trails

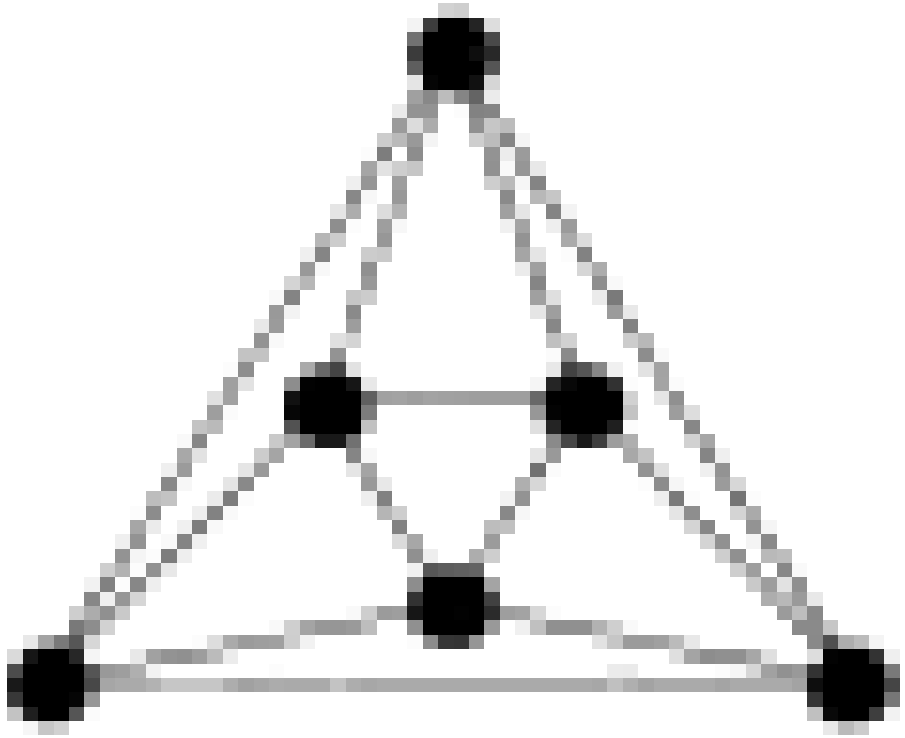
a cycle using each edge just once

An Eulerian trail exists in a graph with all vertices of even degree.

How many extra edges are required to eulerise this graph of a tetrahedron?

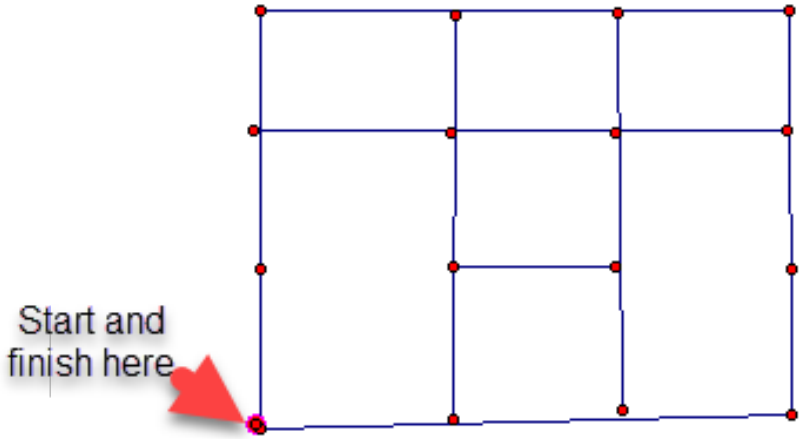


Is there an Eulerian trail on this graph of an octahedron?



Year 9: Circuits Around Networks
Trails and Paths

- How many of the 22 blocks does a bus need to cover twice as it goes all around this network?
- How many blocks does the complete bus route take?
- How many blocks for a garbage truck that needs to cover both sides of every road?
- How many Hamilton circuits are there? (visiting each of the 16 vertices just once)



Circuits Around Networks Solution

These six blocks need to be covered twice.

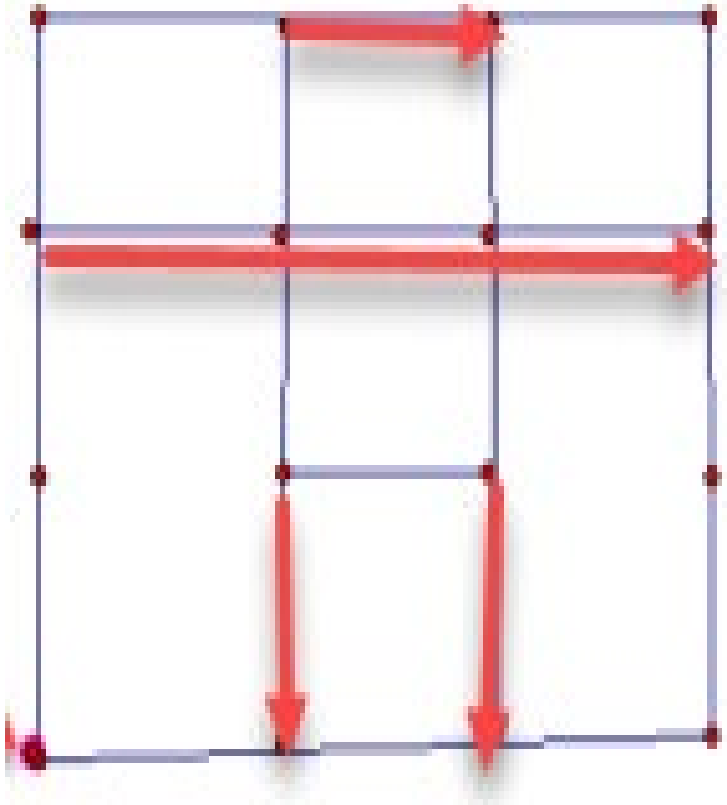
- The other 16 blocks can be covered by an Eulerian trail, so the bus route takes

  6 x 2 + 16 = 28 blocks.

- The garbage truck must also cover 16 blocks in the reverse direction –

  a total of 28 + 16 or 2 x 22 = 44 blocks.

- There are 3 Hamilton circuits here, each in clockwise or anti-clockwise directions.

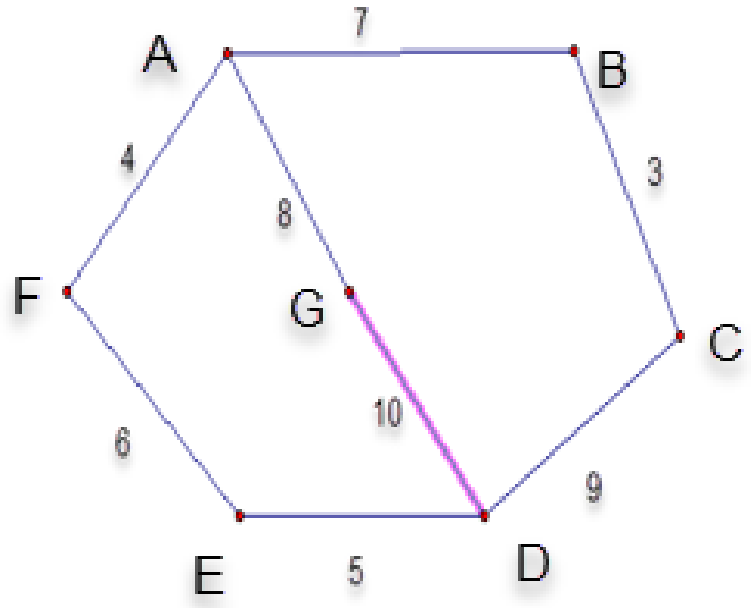


Year 9+: Trees

Trees contain no cycles

Find the **minimum spanning tree** for this diagram showing travelling times between cities.

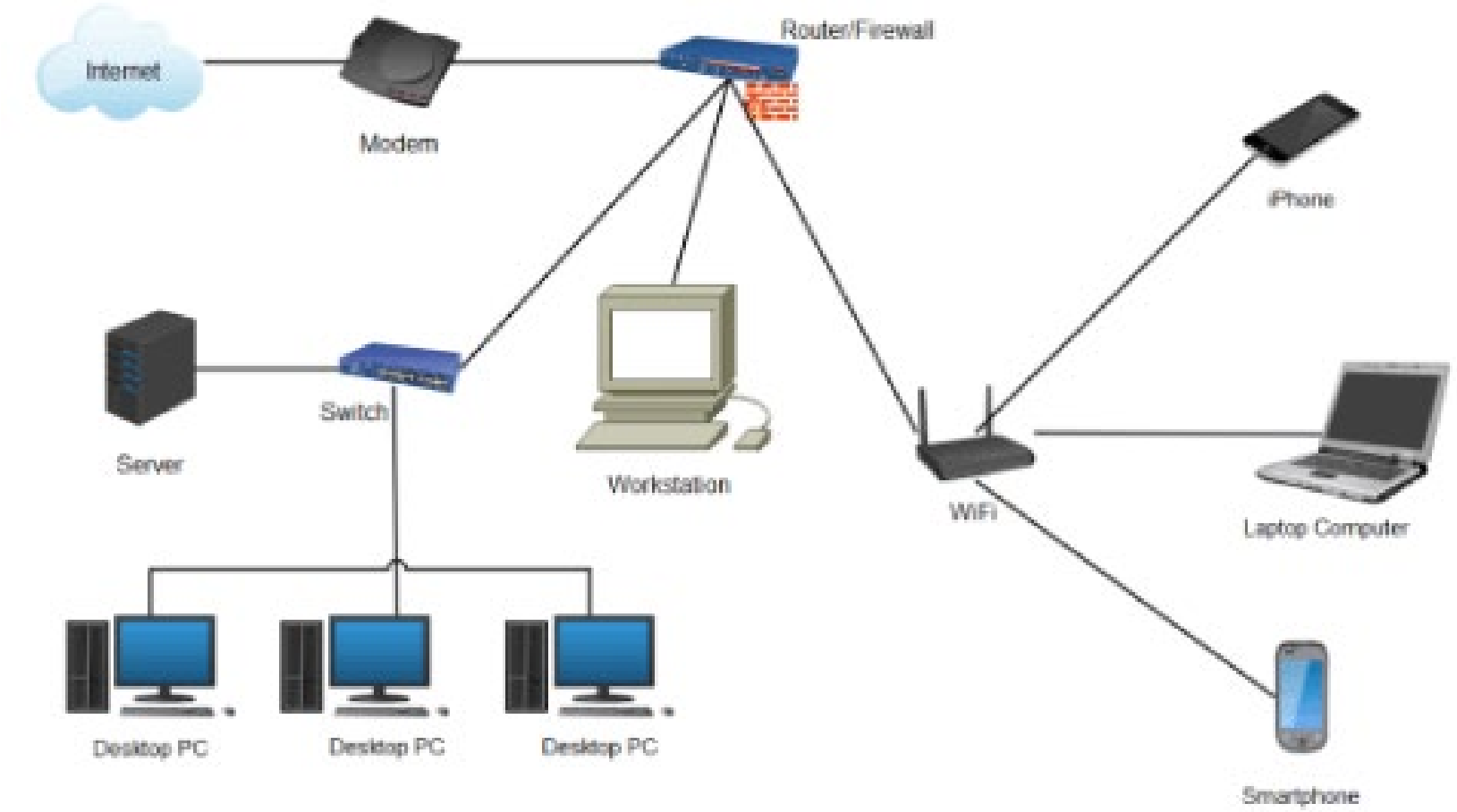
A tree on 7 vertices has 6 edges.
Choose the 6 shortest edges.
(Kruskal's algorithm)



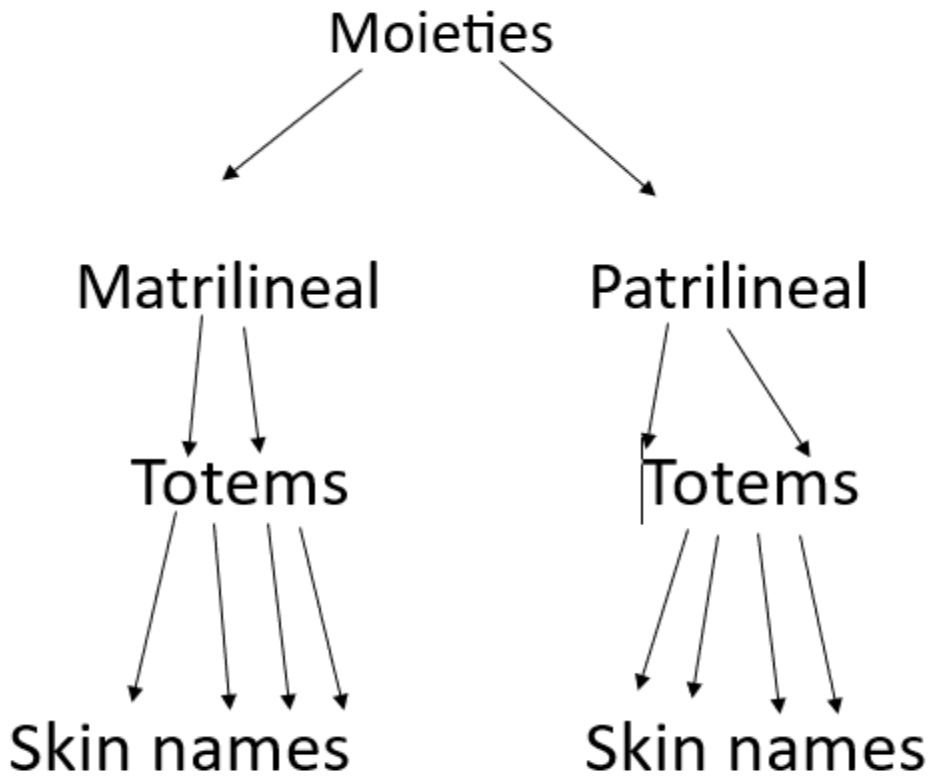
Which one or two cities might be best as distribution centres?

A Local Area Network Tree

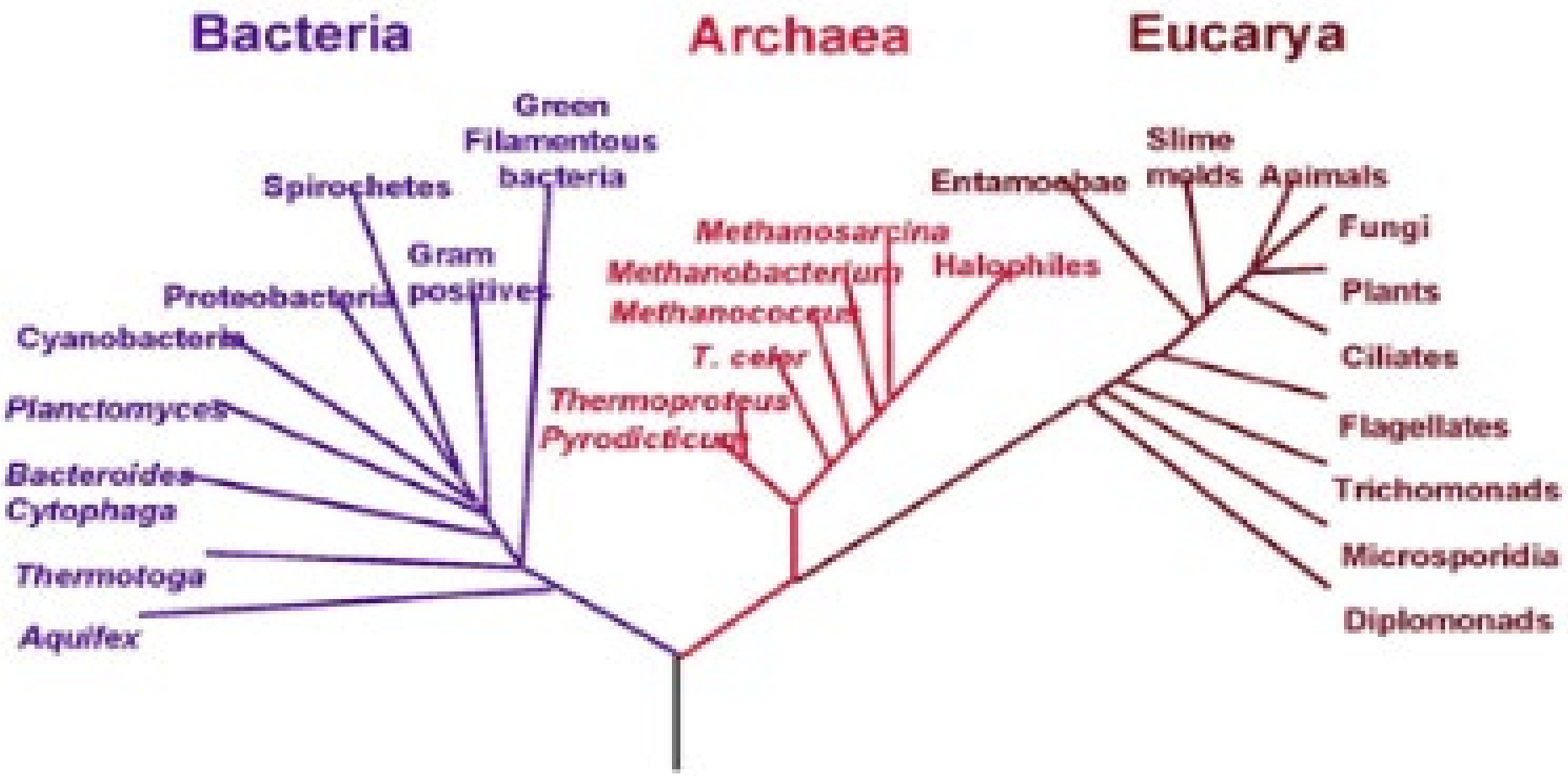
Adapt for your home or school.



An Indigenous Kinship Tree?



Phylogenetic Tree of Life



Year 10: Weighted Graphs

The Postman and Travelling Salesman Problems

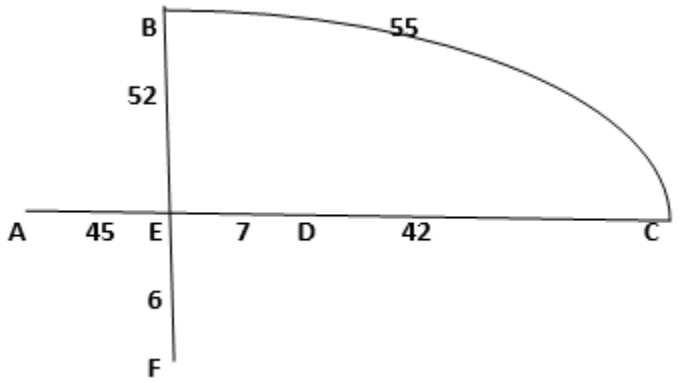
Find the shortest distance for A to visit the other cities and return home.
The postman can have repeat visits. The travelling salesman can't.

distances	Home A	B	C	D	E	F
Home A	0	136	63	111	45	99
B	136	0	**55**	126	52	132
C	63	55	0	42	69	56
D	111	126	42	0	7	141
E	45	52	69	7	0	6
F	99	132	56	141	6	0

The **Guan** (Chinese Postman) Problem

Using the minimum spanning tree

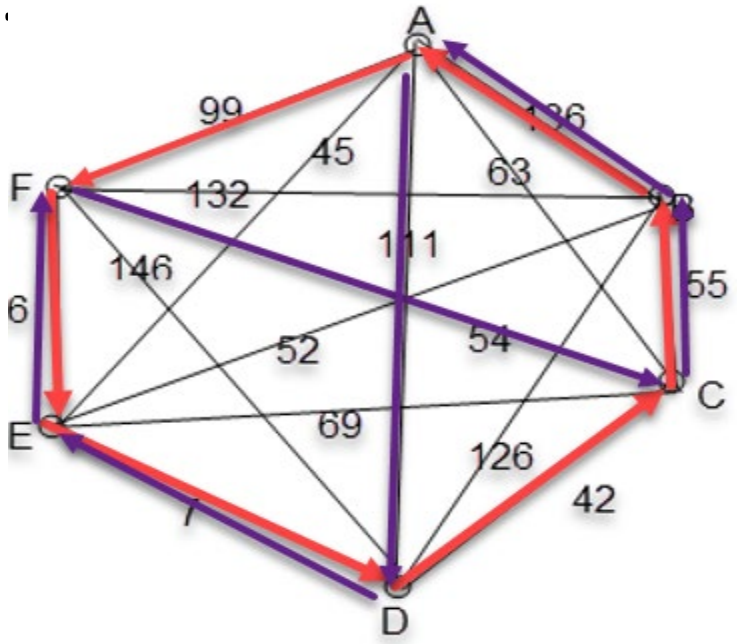
- The graph shows the minimum spanning tree of length 152 plus the edge BC.
- Route AEFEDCBEA uses AE and EF twice and visits E three times.
- Its length is 258.



Travelling Salesman Solution

There are 60 Hamilton circuits that could be tested.

- The minimum spanning tree to all vertices except E (216) plus the two shortest edges from E (6 + 7) gives 229, so the solution must be ≥ 229.
- The nearest neighbour routes have length 369 starting from D and 345 starting from E.



Year 10+: Weighted Directed Graphs

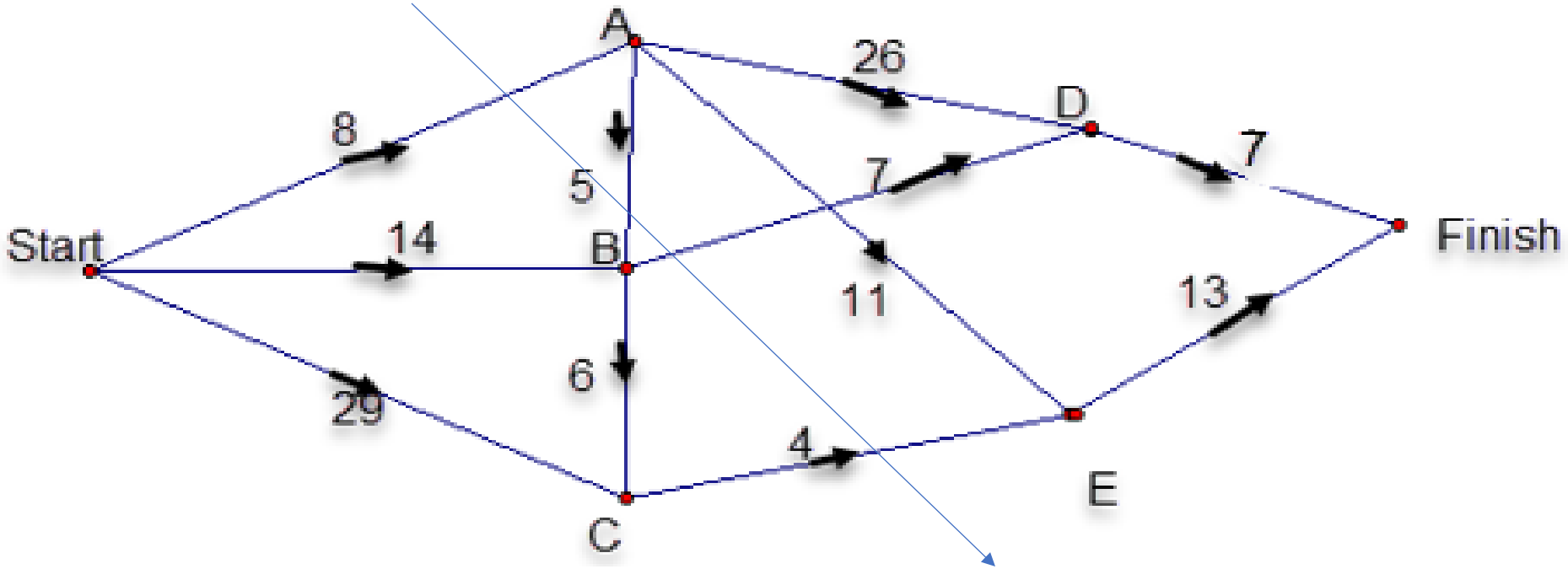
Longest path to D = 8 + 26 = 34

Critical Path: 29 + 4 + 13 = 46.

Slack time at D = (46-7) – 34 = 5

Maximum flow: minimum cut = 8 + 7 + 4 = 19

as shown by the arrow



Summary – for Discussion

How much before Year 10?

	As presented here:-
• Simple and Complete Graphs	Year 6+?
• Directed Graphs	Year 7+?
• Bipartite Graphs	Year 7+?
• Planar Graphs	Year 8+?
• Euler's Rule for Planar Graphs	Year 8+?
• Euler and Hamilton Circuits	Year 9+?
• Trees	Year 9+?
• Weighted Graphs	Year 10?
• Weighted Directed Graphs	Year 10+?

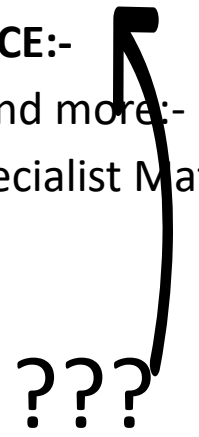
Maths v2.0 Level 10:-

Interpret networks and network diagrams used to represent relationships in practical situations - and describe connectedness.

VCE:-

All of this and more:-

Proof in Specialist Maths 3/4.



The ‘Space’ Curriculum

What to ditch? – add? – move?

Student needs and interests?

Graphs and networks – but at which earlier Year levels?

Congruence axioms? Angles on parallels? Angles in a circle? Euclidean proof?

- Ref: My article planned for Vinculum 1/2025

rob_money22@yahoo.com.au

Some Teaching Resources

- COMAP – USA - Consortium for Maths and its Applications
- Mathigon - USA
- Math Insight – UK
- NRich – UK
- Plus Maths – UK
- The Wilson pdf – UK, for teachers https://www.maths.ed.ac.uk/papers/wilsongraph.pdf

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Be in it to WIN!